

Problem 3.1

Consider a theory with superpotential

$$W = \lambda A_0 + mA_1 A_2 + Y A_0 A_1^2, \quad m^2 > 2\lambda Y.$$

- a) Determine the minimum of the scalar potential V .
- b) Compute the mass spectrum for all bosons and all fermions for $\langle A_0 \rangle = 0$.
- c) Verify the sum rule $\text{Str} M^2 = 0$.

Problem 3.2

Consider a supersymmetric $U(1)$ gauge theory with gauge coupling g and two massive chiral multiplets $\Phi_{\pm}$ of opposite $U(1)$ charge, superpotential $W = m\Phi_+\Phi_-$ and with a non-vanishing FI-term ξ_{FI} .

- a) Give the Lagrangian in superspace and in components using the formulas given in class.
- b) Determine the minimum of V for $|m|^2 > \xi_{\text{FI}}g$. Is the $U(1)$ spontaneously broken?
Is supersymmetry spontaneously broken?
- c) Determine the minimum of V for $|m|^2 < \xi_{\text{FI}}g$. Which symmetries are spontaneously broken? Is there a choice of parameters where supersymmetry is unbroken and the $U(1)$ is broken?

Problem 3.3

- a) Compute all Christoffel symbols and all components of the Riemann curvature tensor for a Kähler manifold with the metric $G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$.
- b) Show

$$[D_i, \bar{D}_{\bar{j}}]v_k = R_{i\bar{j}k}{}^l v_l,$$

$$\text{where } D_{\bar{j}}v_k = \partial_{\bar{j}}v_k, D_i v_k = \partial_i v_k - \Gamma_{ik}^l v_l.$$

- c) Show that the Ricci tensor is given by

$$R_{i\bar{j}} = \partial_i \partial_{\bar{j}} \ln \det(G_{l\bar{k}}).$$