Problem 4.1

Consider the scalar potential of $N = 1$ supergravity in the form

$$V = e^G \big(G^{i\bar{j}} G_i G_{\bar{j}} - 3 \big) \ , \qquad G = K + \ln |W|^2$$

Compute the mass matrices in a Minkowskian background and show

$$M^2_{i\bar{j}} = \langle (\nabla_i G_k \bar{\nabla}_{\bar{j}} G^k - R_{i\bar{j}k\bar{l}} G^k G^{\bar{l}} + G_{i\bar{j}}) e^G \rangle \ , \qquad M^2_{ij} = \langle (G^k \nabla_i \nabla_j G_k + \nabla_i G_j + \nabla_j G_i) e^G \rangle \ ,$$

where $\nabla_i G_j = \partial_i G_j - \Gamma^k_{ij} G_k, \ \nabla_i G_{\bar{j}} = G_{i\bar{j}}$.

Hint: : Use $\langle \nabla_i V \rangle = 0$ as the condition for a Minkowski minimum.

Problem 4.2

The Polonyi model is defined by

$$K = \phi \bar{\phi} \ , \qquad W_P = m^2 (\phi + \beta) \ , \qquad m, \beta \in \mathbb{R}$$

a) For which β is supersymmetry spontaneously broken?

b) Check that $\kappa\phi = \pm(\sqrt{3} - 1)$, $\kappa\beta = \pm(2 - \sqrt{3})$ is a Minkowskian extremum of the potential V.

c) Compute the gravitino mass and $\langle F_\phi \rangle$.

Problem 4.3

Consider the situation where an observable sector is coupled to the Polonyi model with

$$K = \phi \bar{\phi} + Q^I \bar{Q}^I \ , \qquad W = \tfrac{1}{2} \mu_{IJ} \, Q^I Q^J + \tfrac{1}{3} Y_{IJL} \, Q^I Q^J Q^K + W_P(\phi) \ ,$$

where Q^I are the fields of the observable sector, m_{IJ}, Y_{IJL} are constant and W_P is given in problem 4.2.

a) Compute the soft scalar masses and the A and B terms assuming that $\langle F_\phi \rangle$ is the only non-vanishing F-term. Are they universal?

b) Compute the soft gaugino masses for the two cases of a gauge kinetic function $f(\phi)$ and $f =$ constant. Are they universal?

Hint: Use the formulas given in section 10 of the lecture notes.